Topics in Mathematical Science V
Autumn 2023

Course theme. From quiver representations to quasi-hereditary algebras

Preliminary lecture plan.

Week	Date	Topic
1	10-05	Overview. Reminder: modules, representations, simple, indecomposable
2	10-12	Quiver representations, (bounded) path algebras
3	10-19	Jordan-Hölder theorem, Krull-Schmidt property
4	10-26	Tensor-Hom adjunction, projectives, injectives
5	11-02	Exact sequences, extension and Ext-groups
6	11-09	Resolution (optional: Tor-groups)
7	11-16	Trace, reject, co/standard modules
8	11-23	(NO LECTURE)
9	11-30	Revision
10	12-07	Hereditary ideal, quasi-hereditary algebras
11	12-14	Some homological properties of qha
12	12-21	[TBC] (NO LECTURE)
13	12-28	(NO LECTURE)
14	01-04	(NO LECTURE)
15	01-11	[TBC] Revision, homological properties continued
16	01-18	[TBC] Example in algebraic Lie theory / Ringel duality
17	01-25	[TBC] ADR construction as non-commutative resolution of singularity

Time and venue.

- Thursday 13:00–14:30, Grad. School of Mathematics Room 409

Evaluation.

- There will be four sets of homework assignments.
- In each assignment α, the mark of the highest scoring question $m_\alpha \in [0,1]$ will be recorded.
- Four other highest scoring questions in all four assignments $h_1, \ldots, h_4 \in [0,1]$ will also be recorded.
- The final score M is the averaged score (percentage) of the four m_α's along with the four h_i's, i.e.

$$M = \frac{1}{8}\left(\sum_\alpha m_\alpha + \sum_{1 \le i \le 4} h_i\right).$$

- The final grading is determined by the final score $M \in [0,1]$ in percentage as follows:

$$A : 85 \sim 100\%, \quad B : 70 \sim 84\%, \quad C : 50 \sim 69\%, \quad \text{Fail} : 0 \sim 49\%.$$

That is, the minimal marks needed to pass the course is 50% (i.e. $M \ge 0.5$).

Textbooks and references. There is no official textbooks. Lecture notes can be found on my webpage under the Teaching section. Some suggested reference books are given below.

For quiver representations, as well as basic representations theory of finite-dimensional algebras.

[ASS] Assem-Simson-Skowronski - Elements of the representation theory of associative algebras vol. 1

[B] D. J. Benson: *Representations and Cohomology: Volume 1, Basic Representation Theory of Finite Groups and Associative Algebras*, Cambridge Studies in Advanced Mathematics 30, Cambridge University Press 1998
Recommended for reference, but not for learning.

[EH] K. Erdmann and T. Holm: *Algebras and representation theory.* Springer Undergraduate Mathematics Series, Springer International Publishing, 2018

[DW] H. Derksen and J. Weyman: *An introduction to quiver representations*, Gradate Studies in Mathematics 184, AMS, 2017

[Z] A. Zimmermann: *Representation Theory: A Homological Algebra Point of View*, Algebra and Applications 19, Springer 2014

Homological algebra.

- All the books before.

[K] H. Krause: *Homological theory of representations* (available online)

[R] J. J. Rotman: *An introduction to homological algebra*, second edition. Springer

Quasi-hereditary algebra.

[KK] M. Klucznik and S. Koenig: (Lecture notes) Characteristic tilting modules over quasi-hereditary algebras

[K] H. Krause: *Homological theory of representations* (available online)

Contact.

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