Classification of representation-finite self-injective algebras

Aaron Chan

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A = representation-finite self-injective algebras

m-fold mesh algebra (these are self-injective):

Label	Translation quiver	Note
$\Delta^{(m)}$	$\mathbb{Z}\Delta/\langle\tau^m\rangle$	$\Delta = A_n, D_n, E_n$
$B_n^{(m)}$	$\mathbb{Z}A_{2n-1}/\langle\rho\tau^m\rangle$	ρ = reflection in horizontal line
$C_n^{(m)}$	$\mathbb{Z}D_{n+1}/\langle\rho\tau^m\rangle$	ρ = order 2 auto of D_{n+1}
$F_4^{(m)}$	$\mathbb{Z}E_6/\langle\rho\tau^m\rangle$	ρ = order 2 auto of E_6
$G_2^{(m)}$	$\mathbb{Z}D_4/\langle\rho\tau^m\rangle$	ρ = fixed order 3 auto of D_4
$L_n^{(m)}$	$\mathbb{Z}A_{2n}/\langle\rho\tau^m\rangle$	ρ = reflection in central horizontal line, then shift half a unit to the right ($\rho^2 = \tau^{-1}$)

Relation between RFS algebra with m-fold mesh algebras (see Dugas articles):

(1) Standard RFS algebra A of tree class Δ (ADE type) and torsion order 1
$\rightsquigarrow$ preprojective algebra $P(\Delta)$
$\rightsquigarrow$ smash product $P(\Delta)\#k[\mathbb{Z}/m] \cong k(\mathbb{Z}\Delta/\langle\tau^m\rangle)$ the mesh algebra

For torsion order not 1
Use B_{n+1} for (Moebius) A_{2n+1} class, C_{n-1} for D_n order 2 class, (see table above) etc.

(2) Γ be finite stable translation quiver such that $k(\Gamma)$ f.d.
$\rightsquigarrow$ valued graph Δ of generalised Dynkin type A to G_2 and L_n
$\rightsquigarrow$ define Δ' using Δ, this will be in ADE type
$\rightsquigarrow$ one can construct Galois covering $\mathbb{Z}\Delta' \to \Gamma$.

Theorem (see Erdmann-Skrownski paper on CY-dim)
A basic connected not isom to underlying ground ring K
A is symmetric of finite rep-type if and only if one of the following:

1. $T(B)$; B=tilted algebra of Dynkin type A_n, D_n, E_6, E_7, E_8
2. $\widehat{B}/\langle\phi\rangle$;
   B=tilted algebra of Dynkin type A_n
   $\widehat{B}$=repetitive algebra of B
   ϕ proper root of Nakayama automorphism $\nu_{\widehat{B}}$
3. Socle deformation of $\widehat{B}/\langle\phi\rangle$
   B=titled algebra of Dynkin type D_{3s}
   ϕ=root of order 3 of Nakayama auto $\nu_{\widehat{B}}$

(2) = Brauer tree algebra, exceptional multiplicity $m \geq 2$, $n = me$, e=number of edges
(1 - A_n case) = Brauer tree algebra with trivial multiplicity

A can be labelled by triple (Δ, f, t)

sAR-quiver of A is	$\mathbb{Z}\Delta/\Pi$ with	$\Pi = \langle \zeta\tau^{-r} \rangle$, infinite cyclic group
Tree class	Δ is a Dynkin graph/quiver	
Frequency	$f = r/m_\Delta$	($m_\Delta = h_\Delta - 1$ where h_Δ is Coxeter number)
Torsion order	$t = \text{order}(\zeta)$	

Δ	A_n	D_n	E_6	E_7	E_8
m_Δ	n	$2n-3$	11	17	29

Δ	A_n, D_{2n+1}, E_6 $(n \geq 2)$	A_1, D_{2n}, E_7, E_8 $(n \geq 2)$
h^*_Δ	h_Δ	$h_\Delta/2$

Theorem[Dugas]:
A-<u>mod</u> is d-Calabi-Yau for some $d > 0$ $\Leftrightarrow$ $(h^*_\Delta, fm_\Delta = 1)$

In such case:

Δ	A_n, D_{2n+1}, E_6 $(n \geq 2)$	A_1, D_{2n}, E_7, E_8 $(n \geq 2)$
	$d = 2r + 1$	either $2 \mid f$ or char $k = 2$
	$x \equiv -(h^*_\Delta)^{-1} \bmod fm_\Delta$	$d \equiv 1 - (h^*_\Delta)^{-1} \bmod fm_\Delta$

Brauer tree algebra: e=number of edges, $r = n = e, t = 1, f = 1$, $d = 2e - 1$